

銘傳大學 99 學年度研究所碩士班招生考試

應用統計資訊學系碩士班

第四節

線性代數試題

(第 1 頁共 1 頁) (限用答案本作答)

可使用計算機 不可使用計算機

1. If $\{u_1=(1,1,1), u_2=(1,0,-1), u_3=(1,-2,1)\}$ is a set of orthogonal basis (直交基底) for \mathbb{R}^3 and $U=2u_1-u_2, V=-2u_2-u_3$,

a. Find the inner product (內積) of U and V . (5%)

b. Find the angle (角度) between U and V . (5%)

2. Let $A = \begin{bmatrix} 1 & 2 & 3 & -3 \\ 1 & 1 & 2 & 4 \\ 0 & 1 & 2 & -5 \end{bmatrix}$, find a set of basis of the following subspace.

a. Column space of A . (5%)

b. Null space of A . (5%)

3. Let $A = \begin{bmatrix} 3 & 14 & 25 & 36 & 47 \\ 0 & 4 & 15 & 26 & 37 \\ 0 & 0 & 5 & 16 & 27 \\ 0 & 0 & 0 & 6 & 17 \\ 0 & 0 & 0 & 0 & 7 \end{bmatrix}$,

a. Find the eigenvalues of matrix A . (5%)

b. Find the eigenvalues of matrix e^A . (5%)

4. Let $A = \begin{bmatrix} 3 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 3 \end{bmatrix}$,

a. Find the eigenvalues of matrix A . (5%)

b. Find an orthogonal matrix P such that $P^{-1}AP$ is a diagonal matrix. (10%)

c. Find P^{-1} (inverse of P). (5%)

d. Find $A^3 - 4A^2 + A$. (10%)

5. $S = \{a_1=(1,1,c), a_2=(1,-1,2), a_3=(-1,1,1)\}$.

a. Find c such that S forms an orthogonal basis for \mathbb{R}^3 . (5%)

b. Express $(2,4,4)$ as a linear combination of a_1, a_2, a_3 . (10%)

6. Use Gram-Schmidt process to transform the basis $\{(1,1,1), (1,2,3), (1,4,9)\}$ for \mathbb{R}^3 into an orthogonal basis for \mathbb{R}^3 . (10%)

7. Let $A = \{a_1, a_2, a_3\}$ and $B = \{b_1, b_2, b_3\}$ be bases for \mathbb{R}^3 , and let $P = \begin{bmatrix} 2 & -1 & 3 \\ 1 & 1 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ be the

transition matrix from B to A .

a. Express b_1, b_2, b_3 as linear combinations of a_1, a_2, a_3 . (5%)

b. Express a_1, a_2, a_3 as linear combinations of b_1, b_2, b_3 . (10%)

試題完