銘傳大學 98 學年度研究所碩士班招生考試

電腦與通訊工程學系碩士班

第二節

通訊數學(微分方程、線性代數)試題

(第 1 頁共 1 頁)(限用答案本作答)　　□可使用計算機 ☑不可使用計算機

(1) 試解出下列微分方程的通解 (10%)

a) $3y' = y^3 \cdot e^x$　　b) $xy' = \dfrac{\ln x}{y \cdot \cos(y^2)}$

(2) 試求一階線性方程法得 $y'(x) + 2xy = e^{-x^2}$ 之通解 (10%)

(3) 試利用正合微分方程法 (Exact differential equations) 判斷以下之微分方程是否有解，若有請解出其通解 (10%)

a) $x^2 - 4xy + (4xy + 2x) \cdot y' = 0$

b) $e^x \cdot \sin(y) - 2x + (e^x \cdot \cos(y) + 1)y' = 0$

(4) 試解出下列高階微分方程的通解 (20%)

a) $y'' + 3y = -2\sin(\sqrt{3}x)$

b) $x^2y'' - 3xy' - 12y = 8\ln x$

(5) Let $A = \begin{pmatrix} 4 & 2 \\ 3 & 3 \end{pmatrix}$ (30%)

a) Find its eigenvalues

b) Find its two independent eigenvectors

c) Diagnoalizing the matrix A

(6) Let T be a linear transformation from R^3 into R^2 and suppose that

$T\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$, $T\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} -1 \\ 4 \end{pmatrix}$ and $T\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 5 \\ -3 \end{pmatrix}$. Please find $T\begin{pmatrix} 3 \\ -4 \\ 5 \end{pmatrix}$.

(10%)

(7) Let $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$. Please prove that $e^{At} = \begin{pmatrix} e^t & 0 & 0 \\ 0 & e^{2t} & 0 \\ 0 & 0 & e^{3t} \end{pmatrix}$ (10%)

試題完